Non-Hermitian Hamiltonians in decoherence and equilibrium theory

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Abstract

There are many formalisms to describe quantum decoherence. However, many of them give a non general and ad hoc definition of "pointer basis" or "moving preferred basis", and this fact is a problem for the decoherence program. In this paper we will consider quantum systems under a general theoretical framework for decoherence and we will present a tentative definition of the moving preferred basis. These ideas are implemented in a well-known open system model. The obtained decoherence and the relaxation times are defined and compared with those of the literature for the Lee- Friedrichs model.

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Contents

I. INTRODUCTION

From the appearance of quantum mechanics many attempts have been made to recover the laws of the classical mechanics through some classical limit. The more common scheme of this type includes *quantum decoherence*. This process eliminates the terms of interference of the density matrix, that are classically inadmissible, since they prevent the use of a classical (boolean) logic. In addition, decoherence gives a rule to select candidates for classical states.

In this work the decoherence is considered an interaction process between an open quantum system and its environment. This process, called *Environment-Induced Decoherence* (EID) determines case by case which is the privileged basis, usually called *moving preferred basis* where decoherence takes place in a decoherence time t_D that is much smaller than the

relaxation time t_R and it defines certain observables that acquire classical characteristics. This is the orthodox position on the subject [1]. The moving preferred basis was introduced, case by case in several papers (see [2]) in a non systematic way. On the other hand in references [3] and [4] Roland Omnès introduces a rigorous and almost general definition of the moving preferred basis based in a reasonable choice of relevant observables, and other physical considerations. Recently it has become evident that dissipation from system to environment was not a necessary condition for decoherence [2] and the arrival to equilibrium of closed systems was also considered ([5]-[14]). Closed system will be discussed at large elsewhere. In this work we focus our attention on EID, which is a well-known theory, with well established experimental verifications, which makes unnecessary any further explanation.

Non-unitary evolutions are essential to explain and study decoherence phenomena, quantum to classical limit, and final equilibrium. These phenomena appear in the evolution of quantum system, where decoherence time and relaxation time can be defined using non-unitary evolutions, poles theory, and non-Hermitian Hamiltonians. We will consider a closed system U and we will define two subsystems: S, the "proper or open system", and E, the environment. It is well-known that in this case the state of the proper system is obtained from the total density operator by tracing over the environmental degrees of freedom. If we consider the Hermitian Hamiltonian of a composed closed system U and the inner product of the evolved state with any observable we can make its analytical continuation, in the energy variable into the lower complex half-plane, and in general we will find poles. These poles are complex eigenvalues of the non Hermitian Hamiltonian H_{eff} that determines the system evolution. These complex eigenvalues define all the possible non-unitary decaying modes with characteristic decaying times proportional to the inverse of the imaginary part of the poles (see [15]-[22]). From these characteristic times we can deduce the relaxation time is the largest characteristic time and it is related with the pole closest to the real axis. We can also deduce the decoherence time, that turns out to be a function of the imaginary part of the poles and the initial conditions of the system. Moreover, we will introduce a tentative definition of the moving preferred basis. All these definitions are considered in the Lee Friedrichs model.

II. TOWARDS A DEFINITION OF THE MOVING PREFERRED BASIS.

In this section we will try to introduce a very general theory for the moving preferred basis in the case of a general distribution of poles and for *any relevant observable space* $\mathcal{O}_R$. For this purpose it is necessary to consider the coordinates of observables and states in the Hamiltonian basis $\{|\omega\rangle\}$ (i.e. the functions $O(\omega,\omega')$ and $\rho(\omega.\omega')$) endowed with extra analytical properties in order to find the definition of a moving preferred basis.

It is well-known that evolution towards equilibrium has two phases (there also is an initial non exponential Zeno-period which is irrelevant in this paper):

i.- An exponential damping phase that can be described studying the analytical continuation of the Hamiltonian into the complex plane of the energy (see [15]-[22]).

ii.- A final decaying inverse-polynomial in t^{-1} known as the long time evolution or Khalfin effect (see [23], [24]), which is difficult to detect experimentally (see [25]). The power law decay for long times described by the Khalfin effect has no intrinsic parameter. It has no characteristic time scale. Khalfin period is the one where the decaying exponential modes are not more dominant and only inverse powers of time modes remain. We can consider that the time characteristic of this period is infinite (or very long). Instead of using the word "infinite", we will use Khalfin time scale.

These two phases will play an important role in the definition of the moving preferred basis. They can be identified by the theory of analytical continuation of vectors, observables and states. To introduce the main equations we will make a short abstract of papers [15] and [20].

A. Analytic continuations in the bra-ket language.

We begin reviewing the analytical continuation for pure states. Let the Hamiltonian be $H = H_0 + V$ where the free Hamiltonian H_0 satisfies (see [15] or [20])

$$H_0|\omega\rangle = \omega|\omega\rangle,\ \langle\omega|H_0 = \omega\langle\omega|, \quad 0 \leq \omega < \infty \tag{1}$$

and

$$I = \int_0^\infty d\omega|\omega\rangle\langle\omega|,\ \langle\omega|\omega'\rangle = \delta(\omega-\omega') \tag{2}$$

Then

$$H_0 = \int_0^\infty \omega|\omega\rangle\langle\omega| d\omega \tag{3}$$

and

$$H = H_0 + V = \int_0^\infty \omega|\omega\rangle\langle\omega| d\omega + \int_0^\infty d\omega \int_0^\infty d\omega' V_{\omega\omega'}|\omega\rangle\langle\omega'| = \int_0^\infty \omega|\omega^+\rangle\langle\omega^+| d\omega \tag{4}$$

where the $|\omega^+\rangle$ are the eigenvectors of H, that also satisfy eq. (2). The eigen vectors of H are given by the Lippmann-Schwinger equations (see [15]. eq. (12) and (13))

$$\langle\psi|\omega^+\rangle = \langle\psi|\omega\rangle + \langle\psi|\frac{1}{\omega + i0 - H}V|\omega\rangle, \quad \langle\omega^+|\varphi\rangle = \langle\omega|\varphi\rangle + \langle\omega|V\frac{1}{\omega - i0 - H}|\varphi\rangle \tag{5}$$

Let us now endow the function of ω with adequate analytical properties (see [16]). E.g. let us consider that the state $|\varphi\rangle$ (resp. $\langle\psi|$) is such that it does not create poles in the complex extension of $\langle\omega|\varphi\rangle$ (resp. in $\langle\psi|\omega\rangle$) and therefore this function is analytic in the whole complex plane. The physical meaning of this hypothesis is that if the system would be a non interacting one it would never reach equilibrium. Moreover we will consider that the complex extensions of function $\langle\omega^+|\varphi\rangle$ (resp. $\langle\psi|\omega^+\rangle$) is analytic but with just one simple pole at $z_0 = \omega_0 - \frac{i}{2}\gamma_0$, $\gamma_0 > 0$ in the lower halfplane (resp. another pole $z_0^* = \omega_0 + \frac{i}{2}\gamma_0, \gamma_0 > 0$ on the upper halfplane) (see [10] for details). Then in this paper, for the sake of simplicity we will always use a model with just one pole and an integral that corresponds to the Khalfin effect. Then we make an analytic continuation from the positive ω axis to some curve Γ of the lower half-plane.

Then (see [15]. eq. (29)) we can define

$$\begin{aligned}
&\langle\widetilde{f_0}|\varphi\rangle \equiv cont_{\omega'\to z_0}\langle\omega'^+|\varphi\rangle, \quad \langle\psi|f_0\rangle \equiv (-2\pi i) cont_{\omega'\to z_0}(\omega' - z_0)\langle\psi|\omega^+\rangle \\
&\langle\widetilde{f_{z'}}|\varphi\rangle \equiv cont_{\omega'\to z'}\langle\omega'^+|\varphi\rangle, \quad \langle\psi|f_{z'}\rangle \equiv cont_{\omega'\to z}\langle\psi|\omega^+\rangle, \ z' \in \Gamma, \forall\ |\varphi\rangle\ \langle\psi|
\end{aligned} \tag{6}$$

and (see [15]. eq. (31))

$$\begin{aligned}
&\langle\psi|\widetilde{f_0}\rangle \equiv cont_{\omega\to z_0^*}\langle\psi|\omega^+\rangle, \quad \langle f_0|\varphi\rangle \equiv (2\pi i) cont_{\omega'\to z_0^*}(\omega - z_0)\langle\omega^+|\varphi\rangle \\
&\langle\psi|\widetilde{f_{z'}}\rangle \equiv cont_{\omega\to z}\langle\psi|\omega^+\rangle, \quad \langle f_z|\varphi\rangle \equiv cont_{\omega\to z}\langle\omega^+|\varphi\rangle, \ \ z \in \Gamma, \forall\ |\varphi\rangle\ \langle\psi|
\end{aligned} \tag{7}$$

where *cont* means analytic continuation. The tilde in $\langle\widetilde{f_0}|$ is originated in the fact that in the complex extension there is no one-to-one correspondence between bra and kets [15].

Finally it can be proved that (see [15] eq. 1.33 and [20] eq.82)

$$H = z_0|f_0\rangle\langle\widetilde{f_0}| + \int_\Gamma z|f_z\rangle\langle\widetilde{f_z}| dz \tag{8}$$

That is a simple extension of the eigen-decomposition of H to the complex plane with the one-pole term and the integral term that produces the Khalfin effect.

When it is possible to neglect the Khalfin term (i. e. for not extremely long times) the Hamiltonian reads where we have only a complex energy z_0.

$$H_{eff} = z_0|f_0\rangle\langle\widetilde{f_0}| \tag{9}$$

This is the non Hermitian Hamiltonian that determines the evolution of the system far from the Khalfin time scale.

B. Analytical continuation in the observables and states language.

What we have said about the pure states and the Hamiltonian can be rephrased in the case of the states, observables, and the Liouvillian operator L (see a review in [27]). But we prefer to follow the line of [15] and keep the Hamiltonian framework and discuss the analytical continuation of $\langle O\rangle_{\rho(t)}$, that we will also symbolize as $(\rho(t)|O)$. In fact, we know that this scalar is the main character of the play so we will completely study its analytical properties. So let us call

$$|\omega) = |\omega\rangle\langle\omega|, \text{ and } |\omega,\omega') = |\omega\rangle\langle\omega'| \tag{10}$$

Then a generic relevant observable is $O_R \in \mathcal{O}_R$

$$O_R = |O_R) = \int d\omega O(\omega)|\omega) + \int d\omega \int d\omega' O(\omega,\omega')|\omega,\omega') \tag{11}$$

and the generic states is

$$\rho_R = (\rho_R| = \int d\omega \rho(\omega)\widetilde{(\omega|} + \int d\omega \int d\omega' \rho(\omega,\omega')\widetilde{(\omega,\omega'|} \tag{12}$$

where (see also [20] eq. (44) or [15]. eq. (45)).

$$\widetilde{(\omega}|\omega') = \delta(\omega-\omega') \quad \text{and} \quad \widetilde{(\omega,\omega'}|\omega'',\omega''') = \delta(\omega-\omega'')\delta(\omega'-\omega''') \tag{13}$$

Then

$$\widetilde{(\omega}|O_R) = O(\omega),\ \widetilde{(\omega,\omega'}|O_R) = O(\omega,\omega') \tag{14}$$

We will consider the subject as general as possible, i.e. O_R would be any observable such that $O_R \in \mathcal{O}_R$ and ρ_R any state $\rho_R \in \mathcal{O}'_R$. In fact, in the next subsection we will only consider the generic mean value $(\rho_R(t)|O_R)$ for two paradigmatic models below. Model 1 with just one pole and the Khalfin effect and Model 2 with two poles.

C. Model 1. One pole and the Khalfin term:

We will use a formalism for states and observables which has been proposed by the Brussels school (led by Ilya Prigogine) in [26]. It can be proved (cf. ([15]) eq. (67)) that the evolution equation of the mean value $(\rho(t)|O)$ is

$$\langle O_R\rangle_{\rho(t)} = (\rho(t)|O_R) = \int_0^\infty \rho^*(\omega)O(\omega)\,d\omega + \int_0^\infty\int_0^\infty \rho^*(\omega,\omega')O(\omega,\omega')\,e^{i(\omega-\omega')t}\,d\omega d\omega' \quad (15)$$

i.e. this mean value in the case $V \neq 0$ reads

$$(\rho(t)|O_R) = \int d\omega(\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O_R) + \int d\omega\int d\omega' e^{i(\omega-\omega')t}(\rho(0)|\Phi_{\omega\omega'})(\widetilde{\Phi_{\omega\omega'}}|O_R) \quad (16)$$

Where $O(\omega) = (\widetilde{\Phi_\omega}|O_R)$, $O(\omega,\omega') = (\widetilde{\Phi_{\omega\omega'}}|O_R)$, $\rho^*(\omega) = (\rho_R(0)|\Phi_\omega)$, $\rho^*(\omega,\omega') = (\rho_R(0)|\Phi_{\omega\omega'})$. These Φ vectors are defined as

$$|\Phi_\omega) = |\omega^+\rangle\langle\omega^+|,\ |\Phi_{\omega\omega'}) = |\omega^+\rangle\langle\omega^{+\prime}|, \quad (17)$$

and

$$\begin{aligned}(\widetilde{\Phi_{\omega\omega'}}| &= \int d\varepsilon[\langle\omega^+|\varepsilon\rangle\langle\varepsilon|\omega'^+\rangle - \delta(\omega-\varepsilon)\delta(\omega'-\varepsilon)](\widetilde{\varepsilon}| + \int d\varepsilon\int d\varepsilon'\langle\omega^+|\varepsilon\rangle\langle\varepsilon'|\omega'^+\rangle\widetilde{(\varepsilon,\varepsilon'}| \\ (\widetilde{\Phi_\omega}| &= \widetilde{(\omega}| \end{aligned} \quad (18)$$

It should be emphasized that according to definitions (10)-(14), $\widetilde{(\omega}| \neq (|\omega^+\rangle\langle\omega^+|)^\dagger$ and $\widetilde{(\omega\omega'}| \neq (|\omega^+\rangle\langle\omega^{+\prime}|)^\dagger$ in contrast to the case of discrete spectra (see [15] for details). Then, if we endow the functions with analytical properties and there is just one pole z_0 in the lower halfplane, we can prove that (see [15] eq. (70))

$$\begin{aligned}(\rho(t)|O_R) = &\int d\omega(\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O_R) + e^{i(z_0^*-z_0)t}(\rho(0)|\Phi_{00})(\widetilde{\Phi_{00}}|O_R) \\ &+ \int_\Gamma dz' e^{i(z_0^*-z')t}(\rho(0)|\Phi_{0z'})(\widetilde{\Phi_{0z'}}|O_R) + \int_{\Gamma^*} dz e^{i(z-z_0)t}(\rho(0)|\Phi_{0z})(\widetilde{\Phi_{0z}}|O_R) \\ &+ \int_{\Gamma^*} dz\int_\Gamma dz' e^{i(z-z')t}(\rho(0)|\Phi_{zz'})(\widetilde{\Phi_{zz'}}|O_R)\end{aligned} \quad (19)$$

where z_0 is the simple pole in the lower half-plane. $|\Phi_z)$, $(\widetilde{\Phi_z}|$, $|\Phi_{zz'})$, and $(\widetilde{\Phi_{zz'}}|$ can be defined as in the case of eq. (6) and (7). The $|\Phi_z)$, $(\widetilde{\Phi_z}|$, $|\Phi_{zz'})$, and $(\widetilde{\Phi_{zz'}}|$ can be also defined as a simple generalization of the vectors $|f_0\rangle$, $\langle\widetilde{f_0}|$, $|f_z\rangle$, and $\langle\widetilde{f_z}|$ ([15]. eq. (42)).

Therefore we can conclude than the last four terms of equation (19) vanish respectively with characteristic times

$$\frac{1}{\gamma_0}; \frac{2}{\gamma_0}; \frac{2}{\gamma_0}; \infty \tag{20}$$

Let us observe that:

i. The vanishing of the second, third, and fourth therms of eq. (19) is an *exponential decaying* corresponding to the first three terms of eq. (20). This will also be the case in more complicated models with many poles.

ii. The ∞ in eq. (20) means that the evolution of the last term of this equation corresponds to a polynomial in t^{-1} , i. e. to the *Khalfin evolution*. This is a very weak effect detected in 2006 [25]. Therefore if there is a finite number of poles and the curve Γ, is below them, the contribution of the integral along Γ corresponds to the Khalfin effect. A closed system model for Khalfin effect can be found in [28], section 6, and an EID-like model in [29], section 5.

Then as we must have $t_D \ll t_R$ and since from eq. (20) we have just two characteristic times γ_0^{-1} and "∞", the only possible choice is $t_D = \gamma_0^{-1}$ and $t_R = \infty$. In fact, for times $t \gg t_D = \gamma_0^{-1}$, eq. (19) reads

$$(\rho(t)|O_R) = \int d\omega(\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O_R) + \int_{\Gamma^*} dz \int_\Gamma dz' e^{i(z-z')t}(\rho(0)|\Phi_{zz'})(\widetilde{\Phi_{zz'}}|O_R) \tag{21}$$

since where $t \gg t_D = \gamma_0^{-1}$ the pole terms have vanished and we just have the Khalfin term. Let us now diagonalize $\rho(t)$ of the last equation as

$$\rho(t) = \int di \rho_i(t)|i(t)\rangle\langle i(t)| \tag{22}$$

where $\{|i(t)\rangle\}$ is the moving eigenbasis of $\rho(t)$. Now let us define a state $(\rho_P(t)|$, the *preferred state*, such that, *for all times,* it is

$$(\rho_P(t)|O_R) = \int d\omega(\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O) + \int_{\Gamma^*} dz \int_\Gamma dz' e^{i(z-z')t}(\rho(0)|\Phi_{zz'})(\widetilde{\Phi_{zz'}}|O) \tag{23}$$

So $\rho_P(t)$ is a state that evolves in a model with no poles and with just the Khalfin term. The functional $(\rho_P(t)|$ is defined by the inner product $(\rho_P(t)|O_R)$ as follows from the Riezs theorem[1].

It is quite clear that

[1] All these formulas are confirmed by the coincidence of results with other methods: e.g. those used to study a $^{208}Pb(2d_{5/2})$ proton state in a Woods-Saxon potential (see [15] Figure 3).

i. when $t < t_D$, $\rho(t) \neq \rho_P(t)$

ii. when $t \to t_D$, $\rho(t) \to \rho_P(t)$

iii. when $t \gg t_D$, $\rho(t) = \rho_P(t)$

The eigen states of the $\rho_P(t)$ are those that we will choose for the moving decoherence basis. In fact, diagonalizing $\rho_P(t)$ we have

$$\rho_P(t) = \sum_j \rho_j(t) \widetilde{|j(t)\rangle} \widetilde{\langle j(t)|} \tag{24}$$

and when $t \to t_D = \gamma_0^{-1}$ we have that $\rho(t) \to \rho_P(t)$ so from eqs. (22) and (24) we see that the eigenbasis of $\rho(t)$ and $\rho_P(t)$ also converge

$$\{|i(t)\rangle\} \to \{\widetilde{|j(t)\rangle}\} \tag{25}$$

Namely the basis $\{|i(t)\rangle\}$ converges to $\{\widetilde{|j(t)\rangle}\}$ and therefore $\rho(t)$ becomes diagonal in $\{\widetilde{|j(t)\rangle}\}$. Thus $\{\widetilde{|j(t)\rangle}\}$ is our definition for the *moving preferred basis* for this case. Since $\rho(t)$ becomes diagonal in the just defined preferred basis $\{\widetilde{|j(t)\rangle}\}$ when $t \to t_D$ and $t_D = \gamma_0^{-1}$ is really the definition of the decoherence time. In this model the relaxation time t_R corresponds with the Khalfin term, i.e. an extremely long time, so that

$$t_D \ll t_R \tag{26}$$

D. Model 2: Two poles without the Khalfin term.

The Khalfin term is so small (see [25]) that it can be neglected in most of the experimental cases. Then we can eliminate the Khalfin term since it corresponds to extremely long time. In this case the Hamiltonian becomes non-Hermitian as in eqs. (9) and (64). So let us consider the case of two poles z_0 and z_1 (and no relevant Khalfin term) where eq. (19) reads:

$$\begin{aligned}(\rho(t)|O_R) = \int d\omega (\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O_R) + e^{i(z_0^*-z_0)t}(\rho(0)|\Phi_{00})(\widetilde{\Phi_{00}}|O_R) + \\ + e^{i(z_1^*-z_0)t}(\rho(0)|\Phi_{10})(\widetilde{\Phi_{10}}|O_R) + e^{i(z_0^*-z_1)t}(\rho(0)|\Phi_{01})(\widetilde{\Phi_{01}}|O_R) \\ + e^{i(z_1^*-z_0)t}(\rho(0)|\Phi_{11})(\widetilde{\Phi_{11}}|O_R)\end{aligned} \tag{27}$$

where $z_0 = \omega_0 - \frac{i}{2}\gamma_0$, $\gamma_0 > 0$, $z_1 = \omega_1 - \frac{i}{2}\gamma_1$, $\gamma_1 > 0$, and we will also consider that $\gamma_0 \ll \gamma_1$ (see [30] section 3, for details). Then the characteristic times (20) now read

$$\frac{1}{\gamma_0}; \frac{1}{\gamma_1 + \gamma_0} = \frac{1}{\gamma_1 + \gamma_0} \approx \frac{1}{\gamma_1} \tag{28}$$

Then we must choose $t_D = \gamma_1^{-1}$ and $t_R = \gamma_0^{-1}$. Now for times $t \gg t_D = \gamma_1^{-1}$, eq. (21) reads

$$(\rho(t)|O_R) = \int d\omega (\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O_R) + e^{i(z_0^* - z_0)t}(\rho(0)|\Phi_{00})(\widetilde{\Phi_{00}}|O_R) \tag{29}$$

and we can define a state $(\rho_P(t)|$ such that, it would be

$$(\rho_P(t)|O_R) = \int d\omega (\rho(0)|\Phi_\omega)(\widetilde{\Phi_\omega}|O_R) + e^{i(z_0^* - z_0)t}(\rho(0)|\Phi_{00})(\widetilde{\Phi_{00}}|O_R) \tag{30}$$

for *all times*. Repeating the reasoning from eqs. (21) to (25) we can see that, diagonalizing this last equation, as in eq. (24), we obtain the moving preferred basis. Then in this case we see that the relaxation is obtained by an exponential damping (not a Khalfin term) and

$$t_R = \frac{1}{\gamma_0} \gg t_D = \frac{1}{\gamma_1} \tag{31}$$

Again, in this case when $t \to t_D = \gamma_0^{-1}$ we have that $\rho_R(t) \to \rho_P(t)$, and we can conclude that the eigenbasis of $\rho(t)$ and $\rho_P(t)$ also converge as in eq. (25). Namely $\rho(t)$ becomes diagonal in the moving preferred basis in a time t_D.

E. The general case

Let us now consider the general case of a system with $N+1$ poles at $z_i = \omega_i' - i\gamma_i$. In this case it is easy to see that eq. (27) (with no Khalfin term) becomes:

$$(\rho(t)|O_R) = (\rho_*|O_R) + \sum_{i=0}^{N} a_i(t) \exp(-\gamma_i t) = (\rho_{R*}|O_R) + f(t) \tag{32}$$

where $(\rho_*|O_R)$ is the final equilibrium value of $(\rho(t)|O_R)$ and the $a_i(t)$ are oscillating functions. In the most general case the z_i will be placed either at random or following some laws. Anyhow in both cases they can be ordered as[2]

$$\gamma_0 \ll \gamma_1 \ll \gamma_2 \ll ... \tag{33}$$

[2] For simplicity we will only consider the case $\gamma_0 \ll \gamma_1 \ll \gamma_2 \ll ...$Other special cases will be considered elsewhere.

Then if $\gamma_0 \ll \gamma_1$ it is quite clear that the relaxation time is

$$t_R = \frac{1}{\gamma_0} \tag{34}$$

So the relaxation time is defined with no ambiguity. Let us now consider the decoherence time. Really each pole z_i defines a decaying mode with characteristic time $t_i = \gamma_i^{-1}$. Moreover the poles contain the essence of the decaying phenomenon and the definition of the decoherence time depends on their distribution and other data like the initial conditions. In fact, the initial conditions seam essential for the definition of t_D. To introduce these conditions, let us define:

$$f(t) = \sum_{i=0}^{N} a_i(t) e^{-\gamma_i t}, \quad f'(t) = \sum_{i=0}^{N} a_i^{'}(t) e^{-\gamma_i t} - a_i(t)\gamma_i e^{-\gamma_i t} \tag{35}$$

so at $t = 0$ we can write the initial conditions as

$$f(0) = \sum_{i=0}^{N} a_i(0), \qquad f'(0) = \sum_{i=0}^{N} a_i^{'}(0) - \sum_{i=0}^{N} a_i(0)\gamma_i \tag{36}$$

Let us call $f(t) = const. \exp g(t) \sim \exp g(t)$, and let us make a Taylor expansion of $g(t)$ as

$$g(t) = g(0) + g'(0)t + \frac{1}{2} g''(0)t^2 + ... \tag{37}$$

So let us postulate the reasonable hypothesis that the decoherence time is $t_D \ll t_R$. Then, in the period before decoherence that we are considering , precisely $t < t_D \ll t_R$, we have $\frac{t}{t_R} \ll 1$. With this condition we have the approximation:

$$g(t) = g(0) + g'(0)t \tag{38}$$

where

$$g(0) = \log f(0), \qquad g\prime(0) = \frac{f'(0)}{f(0)} \tag{39}$$

These equations contain the initial conditions. Then in this approximation:

$$f(t) = e^{g(0)} e^{tg'(0)} = f(0) \exp\left(\frac{\sum_{i=0}^{N} a_i^{'}}{\sum_{i=0}^{N} a_i} t\right) \exp\left(-\frac{\sum_{i=0}^{N} a_i \gamma_i}{\sum_{i=0}^{N} a_i} t\right) \tag{40}$$

So we define

$$\bar{a}_i(t) = f(0) \exp\left(\frac{\sum_{i=0}^{N} a_i^{'}}{\sum_{i=0}^{N} a_i} t\right) \qquad \text{and} \qquad \gamma_{eff} = \frac{\sum_{i=0}^{N} a_i \gamma_i}{\sum_{i=0}^{N} a_i} \tag{41}$$

And (40) becomes

$$f(t) = \bar{a}_i(t) \exp\left(-\gamma_{eff} t\right) \tag{42}$$

The decoherence time is

$$t_D = \frac{1}{\gamma_{eff}} \tag{43}$$

Then γ_{eff} and t_D are both functions of the initial conditions. We will see that this t_D coincides with the one of the Omnès example in the next subsection.

Let us now consider the definition of the moving preferred basis. It is clear that, for the time $t \gg t_D$, the modes with characteristic times $t_i < t_D$ (i.e. $\gamma_i > \gamma_{eff}$), that we will call the *fast modes*, have become negligible in eq. (32). Then we can define the functional $(\rho_P(t)|$ as

$$(\rho_P(t)|O_R) = (\rho_*|O_R) + \sum_{i=0}^{M} a_i(t) \exp\left(-\gamma_i t\right) \tag{44}$$

where the sum in this equation only contains the $M < N$ poles such that $\gamma_i < \gamma_{eff}$, where the γ_i correspond to the *slow modes*. This is our *adiabatic* choice since we have selected the slow modes of decaying to define $\rho_P(t)$ and rejected the fast modes. Our adiabatic choice corresponds to keep the slow modes and disregard the fast ones. Thus, for us the *robust* modes are the *slow* modes since they are "the less affected by the interaction with the environment", that creates the poles, if compared with the fast modes, and it is usual to say that these robust modes are those that define the moving preferred basis. In fact:

i.- If the Hamiltonian would only be H_0 (cf. eq. (3)) there would not be poles (and this is the usual case in the literature). But the complex extension of the complete Hamiltonian H (cf. eq. (3)) certainly has poles. Therefore the poles are created by the *interaction* Hamiltonian V.

ii.- Thus the slow modes and the fast ones are defined by these poles, and in the case we are considering, i.e. EID, the poles are defined by the *interaction* with the environment.

iii.- Then it is reasonable to call *robust* the slow modes, since the environment interaction has smaller influence in these poles, and we conclude that these are the modes that define the moving preferred basis.

This is our definition of *robustness.* Analogously, if we compute the *linear entropy* we will have a slower variation of this entropy, if we only consider the slow modes, than if we

consider all the modes (including the fast ones). This would be our minimization of the linear entropy: the moving preferred basis evolution only contains the slow modes.

Moreover, when $t \gg t_D$ the motions produced by the fast modes, such that $\gamma_i > \gamma_{eff}$, namely those with motions faster than the one of the evolution of eq. (41), are no more relevant for $\rho(t)$, and $\rho_P(t) \to \rho(t)$. Then we diagonalize $\rho_P(t)$ and we obtain the moving preferred basis $\{\widetilde{|j(t)\rangle}\}$. The only influence in the evolution of $\rho_P(t)$ is given the poles such that $\gamma_i < \gamma_{eff}$. When $t \to t_D$, $\{|i(t)\rangle\} \to \{\widetilde{|j(t)\rangle}\}$ the eigenbasis of $\rho(t)$ where $0 \leq t \leq \infty$. This $\{\widetilde{|j(t)\rangle}\}$ is our candidate for a general definition of moving preferred basis.

III. THE OMNÈS OR LEE-FRIEDRICHS MODEL.

Our more complete and simplest example of decoherence in open systems is the Omnès "pendulum" (i. e. oscillator [31]) in a bath of oscillators, that we will compare with the poles theory in the following subsections. In fact the Omnès model could be considered a poles model if we retain the poles and neglect the Khalfin term. Moreover in the Omnès philosophy the moving preferred basis must be related to some "collective variables" in such a way that they would be experimentally accessible. In this case this variable is the center of mass of the pendulum, i. e. the mean value of the position of a coherent state. In [31] page 285 a one dimensional "pendulum" (the system) in a bath of oscillators (the environment) is considered. Then the Hamiltonian reads

$$H = \omega a^\dagger a + \sum_k \omega_k b_k^\dagger b_k + \sum_k (\lambda_k a^\dagger b_k + \lambda_k^* a b_k^\dagger) \tag{45}$$

where $a^\dagger(a)$ is the creation (annihilation) operator for the system, $b_k^\dagger(b_k)$ are the creation (annihilation) operators for each mode of the environment, ω and ω_k are the energies of the system and of each mode of the environment and λ_k are the interaction coefficients.

Then let us consider a state

$$|\psi(t)\rangle = a|\alpha_1(t)\rangle \prod_k |\beta_{k1}(t)\rangle + b||\alpha_2(t)\rangle \prod_k |\beta_{k2}(t)\rangle \tag{46}$$

where $|\alpha_1(0)\rangle, |\alpha_2(0)\rangle$ are *coherent* states for the "system" corresponding to the operator $a^\dagger$, with center in $x_1(0)$ and $x_2(0)$ respectively, and $|\beta_{k1}(0)\rangle$, $|\beta_{k2}(0)\rangle$ are a coherent state for the environment corresponding to the operator $b_k^\dagger$. Let the initial conditions be

$$|\psi(0)\rangle = a|\alpha_1(0)\ \{\beta_{k1}(0) = 0\}\rangle + b|\alpha_2(t), \{\beta_{k2}(0) = 0\rangle \tag{47}$$

Moreover Omnès shows that, under reasonable hypotheses and approximations the *relaxation time* of the system is

$$t_R = 1/\gamma \tag{48}$$

where

$$\gamma = \pi \int n(\upsilon')d\upsilon'\lambda^2_{\upsilon'}\delta(\omega - \upsilon') \tag{49}$$

where $n(\upsilon')d\upsilon' = d\mathbf{k}$. On the other hand, the decoherence time of the system is (see [31], pp. 289-291)

$$t_D \sim \frac{1}{m\omega_0 L_0^2} t_R \tag{50}$$

where $L_0 = |x_1(0) - x_2(0)|$. In the next subsection, we will attempt to recover these results using the polar technique.

A. The characteristic times from the polar technique.

A particular important model can be studied, like the one in [21], with the Hamiltonian

$$H = \omega_0 a^\dagger a + \int \omega_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}} d\mathbf{k} + \int \lambda_k (a^\dagger b_{\mathbf{k}} + b^\dagger_{\mathbf{k}} a) d\mathbf{k} \tag{51}$$

i.e. a continuous version of (45). In this continuous version we are forced to endow the scalar $(\rho(t)|O_R)$ with some analyticity conditions. Precisely function λ_k (where $k = \omega_k = |\mathbf{k}|$) is chosen in such a way that

$$\eta_\pm(\omega_k) = \omega_k - \omega_0 - \int \frac{d\mathbf{k}\lambda_k^2}{\omega_k - \omega_{k'} \pm i0} \tag{52}$$

does not vanish when $k \in \mathbb{R}^+$, and its analytic extension $\eta_+(z)$ in the lower half plane only has a simple pole at z_0. This fact will have influence on the poles of $(\rho(t)|O_R)$ as in the last section and we know that the study of $(\rho(t)|O_R)$ is the essential way to understand the whole problem.

The Hamiltonian (51) is sometimes called the Lee-Friedrichs Hamiltonian and it is characterized by the fact that it contains different *number of modes sector* (number of particle sectors in QFT). In fact, $a^\dagger$ and $b^\dagger_{\mathbf{k}}$ are creation operators that allow to define these numbers of mode sectors. e. g. the one mode sector will contain states like $a^\dagger|0\rangle$ and $b^\dagger_{\mathbf{k}}|0\rangle$ (where

$a|0\rangle = b_{\mathbf{k}}|0\rangle = 0$). Then the action of $\exp(-Ht)$ (or simple the one of H) will conserve the number of modes of this sector in just one mode, since in (51) all the annihilation operators are preceded by a creation operator. This is also the case for the $n-$mode sector.

1. The Friedrichs model and the relaxation time

In the case of the one mode sector this model is the so called Friedrichs-Fano-Anderson or Friedrichs model. For a complete discussion on this model see [32]. The Hamiltonian of the Friedrichs model is

$$H_F = \omega_0 |1\rangle \langle 1| + \int \omega_k |\omega\rangle \langle \omega| \, d\omega + \int (\lambda(\omega) |\omega\rangle \langle 1| + \lambda^*(\omega) |1\rangle \langle \omega|) \, d\omega \tag{53}$$

(this Hamiltonian which is similar to the one of eq. (4), is expressed just in the variable ω, the one that will be analytically continued). As a consequence of the analyticity condition, mentioned above, this simple Friedrichs model only shows one resonance. In fact, this resonance is produced in z_0. In paper [32] we can see that the poles we compute here are the same as the poles of the Green´s function. Let H_F be the Hamiltonian of the complex extended Friedrichs model, i.e. the Hamiltonian of eq. (8), then[3] :

$$H_F|z_0\rangle = z_0|z_0\rangle, \qquad H_F|z\rangle = z|z\rangle \tag{54}$$

where $z_0 = \omega_0 + \delta\omega_o - i\gamma_0 = \omega_0' - i\gamma_0$ is the only pole and $z \in \Gamma$ corresponds to the integral term and to the Khalfin effect.

The Lee-Friedrichs model, describing the interaction between a quantum oscillator and a scalar field, is extensively analyzed in the literature. Generally, this model is studied by analyzing the one excited mode sector, i.e. the Friedrichs model. Then, if we compute the pole, of this last model, up to the second order in λ_k we obtain that

$$z_0 = \omega_0 + \int \frac{d\mathbf{k}' \lambda_{k'}^2}{\omega_0 - \omega_k + i0} \tag{55}$$

So the pole (that will corresponds to the one closest to the real axis in the Lee-Friedrichs model) can be computed (see [33] eq. (42)). These results coincide (mutatis mutandis) with the one of Omnès book [31] page 288, for the relaxation time. In fact:

$$\frac{1}{\omega_0 - \omega' + i0} = P\left(\frac{1}{\omega_0 - \omega'}\right) - i\pi\delta(\omega_0 - \omega') \tag{56}$$

[3] Only symbolically, since the poles really belong to the scalar $(\rho(t)|O)$, as in the last section.

where P symbolizes the "principal part", so

$$z_0 = \omega_0 + P\int \frac{d\mathbf{k}'\lambda^2_{\mathbf{k}'}}{\omega_0 - \omega_k} - i\pi \int d\mathbf{k}'\lambda^2_{\mathbf{k}'}\delta(\omega_0 - \omega_k) \tag{57}$$

Then if $d\mathbf{k} = n(\omega)d\omega$ we have

$$\delta\omega = P\int \frac{n(\omega')d\omega'\lambda^2_{\omega'}}{\omega_0 - \omega'}, \qquad \gamma = \pi \int n(\omega')d\omega'\lambda^2_{\omega'}\delta(\omega_0 - \omega') \tag{58}$$

where $\delta\omega$ is a shift and γ a damping coefficient, then the system would arrive to a state of equilibrium, namely the results of [31] page 288, and the one contained in eq. (49) yields:

$$z_0 = (\omega_0 + \delta\omega) - i\gamma = \omega_0' - i\gamma \tag{59}$$

So the Omnès result for the relaxation time *coincides*, as we have already said, with the one obtained by the pole theory, precisely (see (48))

$$t_R = \frac{1}{\gamma} \tag{60}$$

2. Other poles of the Lee-Friedrichs model.

Let us now consider the Lee-Friedrichs Hamiltonian (51) for the many modes sectors. Then, as an example for the three mode sector (with just the unique pole z_0 and z_1, z_2, or z_3 "real continuous eigenvalues" transported to the curve Γ) we have:

$$H|z_a, z_b, z_c\rangle = (z_a + z_b + z_c)|z_a, z_b, z_c\rangle \tag{61}$$

where $(z_a + z_b + z_c)$ is the eigenvalue. Then $z_1, z_2, z_3 \in \Gamma$ is the Khalfin terms (i.e. they belong to the complex contour on the lower complex energy plane), and let z_0 be the pole of one particle sector. So in the real complex plane the spectrum of H contains

1.- Eigenvalues $(z_1 + z_2 + z_3)$ with three points of the curve Γ.

2.- Eigenvalues $(z_1 + z_2 + z_0)$, $(z_1 + z_0 + z_3)$ and $(z_0 + z_2 + z_3)$, with two points of the curve Γ and the pole z_0.

3.- Eigenvalues $(z_1 + z_0 + z_0)$, $(z_0 + z_2 + z_0)$ and $(z_0 + z_0 + z_3)$, with a pole at $2z_0$, and one point of the curve Γ.

4.- Eigenvalue $(z_0 + z_0 + z_0)$, with a pole at $3z_0$.

These values appear in expression of the mean value as $\sim e^{-i\frac{z_0}{\hbar}t}$ (like in eq. (19) second term of the l.h.s.) or as $\sim \int_\Gamma e^{-i\frac{z}{\hbar}t}f(z)dz$ (like in eq. (19) three last terms of the l.h.s.). Then we have that the four cases above become:

1.- $\int_\Gamma \int_\Gamma \int_\Gamma e^{-i\frac{(z_1+z_2+z_3)}{\hbar}t} f(z_1,z_2,z_3)dz_1dz_2dz_3$

2.- $\int_\Gamma \int_\Gamma e^{-i\frac{(z_1+z_2+z_0)}{\hbar}t} f(z_1,z_2)dz_1dz_2$, and the same for the combinations $(z_1+z_0+z_3)$, $(z_0+z_2+z_3)$

3.- $\int_\Gamma e^{-i\frac{(z_1+2z_0)}{\hbar}t} f(z_1)dz_1$, and the same for the combinations (z_2-2z_0), (z_3+2z_0)

4.- $e^{-i\frac{3z_0}{\hbar}t}$

Then if we neglect the Khalfin we just have the point 4.

Of course in the general case $3 \to n$ we would have $e^{-i\frac{nz_0}{\hbar}t}$ (for the point n) , plus many integrals on the curve Γ (for the points $1, 2, ...n-1$) corresponding to Khalfin terms Then if we neglect the integrals that produce the Khalfin effect, since this effect corresponds to extremely long times, the Γ term disappears and we simply have a pole at $z_n = nz_0$. This elimination (in the case of just one pole z_0) introduces in the model a structure of a complex oscillator. Then we can introduce a non Hermitian effective Hamiltonian

$$H_{eff} = z_0\left(a_0^\dagger a_0 + \frac{1}{2}\right) = z_0\left(N_0 + \frac{1}{2}\right) \tag{62}$$

where $a_0^\dagger$ and a_0 are creation and annihilation operators and $N_0 = a_0^\dagger\, a_0$ is the number of modes operator and

$$N_0|n\rangle = n|n\rangle \tag{63}$$

In the case of large n, H_{eff} becomes extremely close to

$$H_{eff} = z_0 a_0^\dagger a_0 = z_0 N_0 \tag{64}$$

Moreover we can call

$$z_n = nz_0 = n(\omega_0 - i\gamma_0) \tag{65}$$

and we will find the evolutions

$$\exp(-iH_{eff}t)|n\rangle = \exp(-inz_0t)|n\rangle = \exp(-iz_nt)|n\rangle \tag{66}$$

So, in this approximation, the effective Lee-Friedrichs Hamiltonian H_{eff} simply is a (non Hermitian) version of H with just damping terms. We will below use this structure.

3. The initial conditions

As an initial conditions, $|\alpha_1(0)\rangle$, $|\alpha_2(0)\rangle$, it is possible to choose any linear combination of the elements $\{|n\rangle\}$ where $n = 0, 1..., \infty$. So we can choose coherent states

$$|\alpha_i(0)\rangle = e^{-\frac{|\alpha_i(0)|^2}{2}} \sum_{n=0}^{\infty} \frac{(\alpha_i(0))^n}{\sqrt{n!}} |n\rangle \tag{67}$$

Then let us choose the initial conditions as the sum of two coherent states, namely:

$$|\Phi(0)\rangle = a\,|\alpha_1(0)\rangle + b\,|\alpha_2(0)\rangle \tag{68}$$

Thus the initial state operator is:

$$\rho(0) = |a|^2\,|\alpha_1(0)\rangle\,\langle\alpha_1(0)| + |b|^2\,|\alpha_2(0)\rangle\,\langle\alpha_2(0)| + ab^*\,|\alpha_1(0)\rangle\,\langle\alpha_2(0)| + a^*b\,|\alpha_2(0)\rangle\,\langle\alpha_1(0)| \tag{69}$$

We choose the two Gaussian (67) with center at $p_{1,2}(0) = 0$, (see [31] eq. (7.15) page 284) and

$$\alpha_1(0) = \frac{m\omega}{\sqrt{2m\omega}} x_1(0) \qquad \text{and} \qquad \alpha_2(0) = \frac{m\omega}{\sqrt{2m\omega}} x_2(0) \tag{70}$$

So $\alpha_1(0)$ and $\alpha_2(0)$ are real numbers. With a change of coordinates we can choose $x_1(0)$ and $x_2(0)$ without loss of generality. So we can consider that the $\alpha_1(0)$ and $\alpha_2(0)$ are both positive. For this reason we will interchange $\alpha_i(0)$ and $|\alpha_i(0)|$ below. With no lost of generality we can choose

$$\alpha_1(0) = 0 \qquad \text{and} \qquad \alpha_2(0) = \frac{m\omega}{\sqrt{2m\omega}} L_0 \tag{71}$$

The macroscopic case

It is easy to prove that for macroscopic initial conditions, i.e. when the peaks of the two Gaussians are far from each other, that is to say $|\alpha_1(0) - \alpha_2(0)| \to \infty$, the states $\{|\alpha_1(0)\rangle, |\alpha_2(0)\rangle\}$ are quasi-orthogonal basis

$$\langle\alpha_1(0)|\alpha_2(0)\rangle \cong \langle\alpha_2(0)|\alpha_1(0)\rangle \cong e^{-\frac{(\alpha_1(0)-\alpha_2(0))^2}{2}} \cong 0 \tag{72}$$

Then, the macroscopic condition for the initial conditions is $|\alpha_1(0) - \alpha_2(0)| \gg 1$. So we have $|\alpha_1(0) - \alpha_2(0)| = \alpha_2(0) \gg 1$,

$$\frac{m\omega}{\sqrt{2m\omega}} L_0 \gg 1 \tag{73}$$

4. Components of the non-diagonal part of the state

Therefore the evolved state is

$$\rho(t) = |a|^2 |\alpha_1(t)\rangle \langle\alpha_1(t)| + |b|^2 |\alpha_2(t)\rangle \langle\alpha_2(t)| + ab^* |\alpha_1(t)\rangle \langle\alpha_2(t)| + a^*b |\alpha_2(t)\rangle \langle\alpha_1(t)| \quad (74)$$

Let us not consider the non-diagonal part of $\rho(t)$, $\rho^{(ND)}(t)$ in the basis of the initial conditions $\{|\alpha_1(0)\rangle, |\alpha_2(0)\rangle\}$. Then we have

$$\rho^{(ND)}(t) = \rho_{12}^{(ND)}(t) |\alpha_1(0)\rangle \langle\alpha_2(0)| + \rho_{21}^{(ND)}(t) |\alpha_2(0)\rangle \langle\alpha_1(0)| \quad (75)$$

Since the basis $\{|\alpha_1(0)\rangle, |\alpha_2(0)\rangle\}$ is quasi-orthogonal, from eq. (74) we have

$$\rho_{ij}^{(ND)}(t) = ab^* \langle\alpha_i(0)|\alpha_1(t)\rangle \langle\alpha_2(t)|\alpha_j(0)\rangle + a^*b \langle\alpha_i(0)|\alpha_2(t)\rangle \langle\alpha_1(t)|\alpha_j(0)\rangle \quad (76)$$

If we consider the evolution given by the non Hermitian Hamiltonian

$$|\alpha_i(t)\rangle = e^{-iH_{eff}t} |\alpha_i(0)\rangle = e^{-\frac{|\alpha_i(0)|^2}{2}} \sum_{n=0}^{\infty} \frac{(\alpha_i(0))^n}{\sqrt{n!}} e^{-iz_n t} |n\rangle \quad (77)$$

we can compute these products $\langle\alpha_i(0)|\alpha_j(t)\rangle$ and we can replace them in (76) to obtain

$$\begin{aligned} \rho_{12}^{(ND)}(t) &\cong ab^* e^{-|\alpha_2(0)|^2\left(1-e^{iz_0^* t}\right)} \\ \rho_{21}^{(ND)}(t) &\cong a^*b e^{-|\alpha_2(0)|^2\left(1-e^{-iz_0 t}\right)} \end{aligned} \quad (78)$$

5. Decoherence time

Since the contributions γ_n of individual poles z_n do not appear explicitly in the equation (78), we may think that such poles are not involved in the outcome.

However, if we express the exponential of (78) as its Taylor series, we have

$$\begin{aligned} \rho_{12}^{(ND)}(t) &\cong ab^* e^{-|\alpha_2(0)|^2} \sum_{n=0}^{\infty} \frac{\left(|\alpha_2(0)|^2\right)^n}{n!} e^{iz_n^* t} = ab^* \sum_{n=0}^{\infty} c_n(t) e^{-\gamma_n t} \\ \rho_{21}^{(ND)}(t) &\cong a^*b e^{-|\alpha_2(0)|^2} \sum_{n=0}^{\infty} \frac{\left(|\alpha_2(0)|^2\right)^n}{n!} e^{-iz_n t} = a^*b \sum_{n=0}^{\infty} c_n^*(t) e^{-\gamma_n t} \end{aligned} \quad (79)$$

where

$$c_n(t) = e^{-|\alpha_2(0)|^2} \frac{\left(|\alpha_2(0)|^2\right)^n}{n!} e^{i\omega_n t} \quad (80)$$

and these equations show that all the diagonal terms vanish when $t \to \infty$ showing that there is decoherence. Now we would like to know the decoherence time, then we must find γ_{eff}. So we analyze the decay of

$$\left|\rho_{12}^{(ND)}(t)\right|^2 = \rho_{12}^{(ND)}(t)\left(\rho_{12}^{(ND)}(t)\right)^* = |ab|^2 e^{g(t)} \tag{81}$$

where

$$g(t) = \ln\left(\left(\sum_{n=0}^{\infty} c_n(t)e^{-\gamma_n t}\right)\left(\sum_{j=0}^{\infty} c_n^*(t)e^{-\gamma_n t}\right)\right) \tag{82}$$

Let us now expand $e^{g(t)}$ as:

$$e^{g(t)} = e^{g(0)+g'(0)t+\frac{1}{2}g''(0)t^2+\ldots} \tag{83}$$

As the decoherence time is a very short one $t_D \ll t_R$ let us neglect it from the quadratic term. Now, from eq. (82) we have that

$$\begin{aligned} g(0) &= 0 \\ g'(0) &= -2e^{-|\alpha_2(0)|^2}\sum_{n=0}^{\infty}\frac{\left(|\alpha_2(0)|^2\right)^n}{n!}\gamma_n = \gamma_{eff} \end{aligned} \tag{84}$$

then from (81) and (84), we have

$$\left|\rho_{12}^{(ND)}(t)\right| \cong |ab|\exp\left(-\gamma_{eff}t\right) \tag{85}$$

This is precisely the interpolation that corresponds to eq. (42). Now we have the decoherence time

$$t_D = \frac{1}{\gamma_{eff}} = \frac{2}{m\omega}\frac{1}{L_0^2}t_R \tag{86}$$

In fact this t_D^{-1} turns out to be a weighted average of the imaginary part of the poles z_n. The same time was found by Omnès in [31] (or in eq. (50) of this paper) and corresponds to the definition (43) of the last section. So in fact, we have found the same result. Also in [31] the result for t_D is only valid for small t as in the last section. So the coincidence of both formalisms is proved.

Let us now consider the mathematical definition of moving preferred basis. The basis $\{|\alpha_1(t)\rangle, |\alpha_2(t)\rangle\} = \{|\alpha_i(t)\rangle\}$, is orthonormal when $L \to \infty$ and the reasoning below is done under this condition. Then let us diagonalize $\rho_P(0)$ (always when $L \to \infty$) as

$$\rho_P(t) = \sum_{i=1,2}\rho_i(t)|\widetilde{i(t)}\rangle\langle\widetilde{i(t)}| = \rho_1(t)|\widetilde{1(t)}\rangle\langle\widetilde{1(t)}| + \rho_2(t)|\widetilde{2(t)}\rangle\langle\widetilde{2(t)}| \tag{87}$$

where $\{|\widetilde{i(t)}\rangle\}$ is our orthogonal moving pointer basis. But for times $t \gg t_D$ $\rho_R(t) = \rho(t)^{(D)}$ we have

$$\rho_P(t) = \rho^{(D)}(t) = |a|^2 |\alpha_1(t)\rangle \langle\alpha_1(t)| + |b|^2 |\alpha_2(t)\rangle \langle\alpha_2(t)| \tag{88}$$

Then, since a linear orthonormal decomposition is unique we find the moving pointer basis

$$|\widetilde{1(t)}\rangle = |\alpha_1(t)\rangle\,, \quad |\widetilde{2(t)}\rangle = |\alpha_2(t)\rangle \tag{89}$$

Then Omnès basis coincides with $\{|\widetilde{1(t)}\rangle, |\widetilde{2(t)}\rangle\}$, but this also is our moving preferred basis since it evolves under the slow motion pole evolution. So Omnès basis and our basis coincide (always when $L \to \infty$).

So we have proved that all the characters of the Omnès model: t_R, t_D, and the moving preferred basis, coincides with our definitions of the last section.

IV. CONCLUSIONS

In this work we have:

i.- Discussed a general scheme for decoherence, that in principle could be used in many examples.

ii.- We have given a quite general definition of moving preferred basis $\{|\widetilde{j(t)}\rangle\}$, and of relaxation and decoherence times for a generic system.

iii.- We have proved that our definitions coincide with those of the Omnès model.

We hope that these general results will produce some light in the general problem of decoherence.

The Omnès formalism, of references [3], [4], and [31] contain the most general definition of moving preferred basis of the literature on the subject. Our basis has another conceptual frame: the catalogue of decaying modes in the non-unitary evolution of a quantum system. But since the Omnès formalism is the best available it would be very important for us to show, in the future, the coincidence of both formalisms, as we have at least done for one model in this paper.

Of course we are fully aware that, to prove our proposal, more examples must be added, as we will do elsewhere. But we also believe that we have a good point of depart. In fact, probably the coincidences that we have found in the Omnès model could be a general feature of the decoherence phenomenon. Essentially because, being the poles catalogue the one that

contains *all the possible decaying modes* of the non unitary evolutions, since relaxation and decoherence are non-unitary evolutions, necessarily they must be contained in this catalogue.

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[1] J. Bub, *Interpreting the Quantum World*, Cambridge University Press, Cambridge, 1997.

[2] M. Schlosshauer, *Decoherence and the Quantum-to-Classical transition*, Springer, Berlin, 2007.

[3] R. Omnès, *Phys. Rev. A* **56**, 3383–3394 (1997).

R. Omnès, *Phys. Rev. A* **65**, 052119 (2002).

[4] R. Omnès, *Understanding quantum mechanics,* Princeton Univ. Press, Princeton, 1999.

[5] R. Gambini and J. Pulin, *Found. of Phys.*, **37**, 7, 2007.

R. Gambini, R. A. Porto and J. Pulin, *Gen. Rel. Grav.*, **39**, 8, 2007.

R. Gambini and J. Pulin, "Modern space-time and undecidability", in V. Petkov (ed.), *Fundamental Theories of Physics (Minkowski Spacetime: A Hundred Years Later)*, Vol. 165, Springer, Heidelberg, 2010.

[6] G.W. Ford, R.F. O'Connell, *Physics Letters A*, **286** (2001) 87–90.

[7] G. Casati and T. Prosen, *Phys. Rev. A* **72**, 032111 (2005).

[8] M. Castagnino*, Int. J. Theor. Phys.,* **38**, 1333, 1999.

M. Castagnino and R. Laura, *Phys. Rev. A* **62**, 022107 (2000).

M. Castagnino and A. Ordoñez, *Int. J. Theor. Phys.*, **43**, 695, 2004.

G. Murgida and M. Castagnino, *Physica A*, **381**, 170, 2007.

[9] M. Castagnino and S. Fortin, *Int J Theor Phys* (2011) **50**:2259–2267.

[10] M. Castagnino and O. Lombardi, *Phys. Rev. A* **72**, 012102 (2005).

[11] O. Lombardi and M. Castagnino, *Studies in History and Philosophy of Modern Physics*, **39**, 380, 2008.

[12] M. Castagnino, *Physics Letters A* **357** (2006) 97–100.

[13] Mario Castagnino et al 2010 *J. Phys. A: Math. Theor.* **43** 065304.

[14] M. Castagnino and O. Lombardi, *Studies in History and Philosophy of Modern Physics*, **35**, 73, 2004.

[15] M. Castagnino, R. Id Betan, R. Laura and R. J. Liotta, *J. Phys. A: Math. Gen.*, **35**, 6055-6074, 2002.

[16] A. Bohm, *Quantum mechanics, foundations and applications,*Springer Verlag, Berlin, 1986.

[17] R. Weder, *Jour. Math. Phys.,* **15,** 20, 1974.

[18] E. Sudarsham, C. Chiu and V. Gorini, *Phys. Rev. D* **18**, 2914–2929 (1978).

[19] M. Castagnino and R. Laura, *Phys. Rev. A* **56**, 108–119 (1997).

[20] R. Laura and M. Castagnino, *Phys. Rev. A* **57**, 4140–4152 (1998).

[21] R. Laura and M. Castagnino, *Phys. Rev. E* **57**, 3948–3961 (1998).

[22] M. Castagnino and S. Fortin, *Modern Physics Letters A*, Vol. **26**, No. 31 (2011) 2365-2373.

[23] K. Urbanowski, *Eur. Phys. J. D*, **54**, 25, 2009.

K. Urbanowski and J. Piskorski, "Energy of unstable states at long times", in A. I. Studenikin (ed.), *Particle Physics at the Year of Astronomy: Proceedings of the Fourteenth Lomonosov Conference on Elementary Particle Physics*, World Scientific Publishing Co., Singapore, 2011.

[24] N. Bleistein and R. Handelsman, *Asymptotic expansion of integrals,* Dover Inc., New York, 1986.

[25] C. Rothe, S. I. Hintschich, and A. P. Monkman, *Phys. Rev. Lett.* 96, 163601 (2006).

[26] T. Petrosky, I. Prigogine and S. Tasaki, *Physica A*, **173**, 175-242, 1991.

T. Petrosky and I. Prigogine, *Physica A*, **175**, 146-209, 1991.

I. Antoniou and I. Prigogine, *Physica A*, **192**, 443-464, 1993.

I.E. Antoniou, Z. Suchenecki, R. Laura and S. Tasaki, *Physica A*, **241**, 737-772, 1997.

[27] M. Castagnino, F. Gailoli and E. Gunzig, *Found. Cosmic Phys.,* **16***,* 221-375, 1996.

[28] F. Gaioli, E. García-Álvarez and J. Guevara, *Int. J. Theor. Phys.,* **36,** 2167-2219, 1997.

[29] D. Arbó, M. Castagnino, F. Gaioli and S. Iguri, *Physica A,* **274,** 469-495**.**

[30] M. Gadella and G. Pronko, *Fortschritte der Physik* **59**, 795-859 (2011).

[31] R. Omnès, *The Interpretation of Quantum Mechanics,* Princeton University Press, Princeton, 1994.

[32] Longhi, S., *Physical Review B* **80**, 165125 (2009).

[33] R. Laura, M. Castagnino and R. Id Betan, *Physica A,* **271,** 357-386, 1999.